Important Problems on L-S / J-J couplings

(1) Prove that the no. of terms (Spectral) for 'pd' configuration is the same for both L-S and J-J couplings.

s	p	d	f
$l=0$	1	2	3

Ans –

p: $l_1 = 1$, $s_1 = \frac{1}{2}$

d: $l_2 = 2$, $s_2 = \frac{1}{2}$

(a) L-S coupling

$$L = l_2 \pm l_1 = 2 \pm 1 = 3\,2\,1$$
$$S = s_1 \pm s_2 = \tfrac{1}{2} \pm \tfrac{1}{2} = 0, 1$$

i) $L = 3 \quad S = 0 \Rightarrow J = L \pm S = 3 \pm 0 = 3$

ii) $L = 2 \quad S = 0 \Rightarrow J = L \pm S = 2 \pm 0 = 2$

iii) $L = 1 \quad S = 0 \Rightarrow J = L \pm S = 1 \pm 0 = 1$

MF $= 2S+1 = 2\times0+1 = 1$

iv) $L = 3 \quad S = 1 \Rightarrow J = L \pm S = 3 \pm 1 = 4 \text{ to } 2,\ 4\,3\,2$

v) $L = 2 \quad S = 1 \Rightarrow J = L \pm S = 2 \pm 1 = 3\,2\,1$

vi) $L = 1 \quad S = 1 \Rightarrow J = L \pm S = 1 \pm 1 = 2\,1\,0$

MF $= 2S+1 = 3$

Terms – i) ${}^1F_3$ ii) ${}^1D_2$ iii) ${}^1P_1$

iv) ${}^3F_{432}$ v) ${}^3D_{321}$ vi) ${}^3P_{012}$

Total spectral term $= 12$

M.F. = multiplicity factor $= (2S+1)$

(b) j-j Couplings

p	d
$l_1 = 1$	$l_2 = 2$
$s_1 = \frac{1}{2}$	$s_2 = \frac{1}{2}$
$J_1 = l_1 \pm s_1$	$J_2 = l_2 \pm s_2$
$= 1 \pm \frac{1}{2} = \frac{3}{2}, \frac{1}{2}$	$= 2 \pm \frac{1}{2}$
	$= \frac{5}{2}, \frac{3}{2}$

i) $J_1 = \frac{3}{2}$, $J_2 = \frac{5}{2}$

ii) $J_1 = \frac{3}{2}$, $J_2 = \frac{3}{2}$

iii) $J_1 = \frac{1}{2}$ $J_2 = \frac{5}{2}$

iv) $J_1 = \frac{1}{2}$ $J_2 = \frac{3}{2}$

i) $J = J_1 \pm J_2 = \frac{3}{2} \pm \frac{5}{2} = \frac{5}{2} \pm \frac{3}{2}$ $= 4$ to 1 4 3 2 1

ii) $J = J_1 \pm J_2 = \frac{3}{2} \pm \frac{3}{2} = 3$ to 0 3 2 1 0

iii) $J = J_1 \pm J_2 = \frac{1}{2} \pm \frac{5}{2} = \frac{5}{2} \pm \frac{1}{2}$ $= 3$ to 2 3 2

iv) $J = J_1 \pm J_2 = \frac{1}{2} \pm \frac{3}{2} = \frac{3}{2} \pm \frac{1}{2}$ 2 to 1 2 1

Total Spectral term = 12

"Also solve similar problems"